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Pull-out in middle of book: **Full Practice Test**

(numbered separately, **after page 76**)



GRAPHS, EQUATIONS, TABLES

D. Equations

Practice Questions

Try these two questions on your own:

1. A bowling alley charges a \$10 fee plus \$15 for every game played.
If you and your friends spent \$55 all together, how many games did you play?

a) Create the equation after assigning letters to each of the items.

b) **I**nsert the known values:

c) Isolate the unknown and **S**olve:

d) **T**ell your answer in words:

e) Create a graph to show the same information:

f) What kind of variation does it show? _____

Answers on page 95-96.

GRAPHS, EQUATIONS, TABLES

D. Equations

The content of the following question (about renting a banquet hall) is not part of the experience of most Grade 9 students. Dr. Don has included this question for practice, because some problems on the Math Test do include content that is unfamiliar to most students.

2. A company is planning a holiday party for its employees. A banquet hall charges a fixed fee for rental of the hall, plus \$15 for each person who attends the party. For a party with 50 people, the total cost is \$1200.

Which equation below models this relation, where C is the total cost in dollars, and n is the number of people attending?

- a) $C = 24n$
- b) $C = 24n + 1200$
- c) $C = 24n + 450$
- d) $C = 15n + x$

Answers and explanations for page 94-95

1. a) Let C = total cost [You are told this value: \$55.]
 f = flat fee [You are told this value: \$10.]
 n = number of games ** This is the stated question: "How many games..."
 g = cost per game [You are told this value: \$15.]

Equation: $C = f + ng$ [total cost = flat fee plus number of games x cost per game]

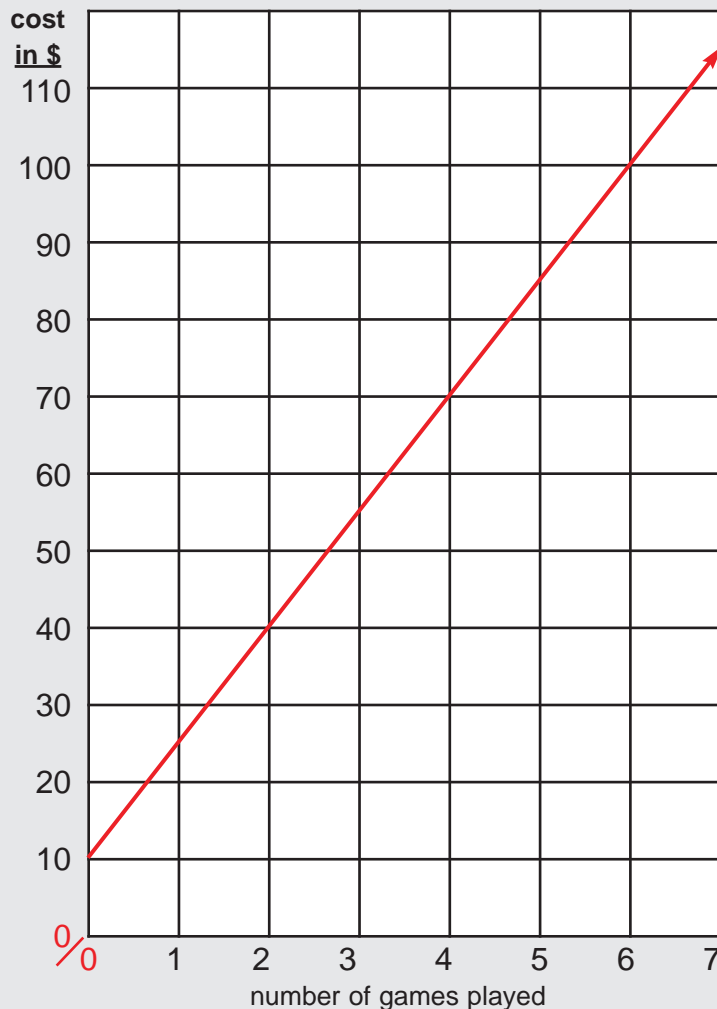
- b) Insert known values: $55 = 10 + (n \times 15)$
- c) Solve
- $$55 = 10 + 15n$$
- $$55 - 10 = 10 - 10 + 15n$$
- $$45 = 15n$$
- $$45 \div 15 = 15n \div 15$$
- $$3 = n$$
- d) Tell answer in words: they played 3 games.
- e) Graph on next page
- f) Partial linear variation

continued on next page...

GRAPHS, EQUATIONS, TABLES

D. Equations

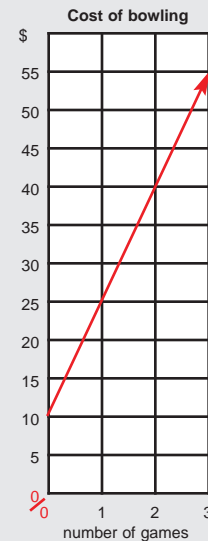
e) **Cost of bowling**



Drawing a grid for a graph can be difficult. You have to decide what to show on the x axis (horizontal) and y axis (vertical). Always keep the intervals constant. On the y axis, the dollars increase by \$10 at every step (or \$5 in the graph below).

The question only asked about the cost of playing three games. The graph on the left would show the cost of up to seven games.

If you made a graph showing only three games, and showing intervals of \$5 in the y axis, your graph could be perfectly correct, as below.



2. Choose answer d).

$C = 15n + x$ means: Total cost = \$15 x the number of people attending, plus the fixed fee for renting the hall (which you don't know, so call the fixed fee x .)

When you insert the values you know ($C = \$1200$, $n = 50$), you can solve the equation to find x .

$$1200 = (15 \times 50) + x$$

$$1200 = 750 + x$$

$$1200 - 750 = 750 - 750 + x \quad (\text{Isolate the unknown})$$

$$450 = x$$

Tell the answer in words: the flat fee for renting the hall is \$450.

[You weren't asked to calculate this, but if you're interested, that's how to solve the equation.]



Guided Review

Fixed and Variable Values

In many of the previous questions, you may have noticed that there was a flat fee or an initial membership cost that resulted in a starting point higher than zero. These values didn't change, no matter how many lawns you cut or visits you made to the gym. They were **fixed values**.

For example, if you wanted to buy some books from an internet site, and they charged you a flat fee of \$5 for shipping, you would have to pay \$5 for shipping whether you bought one book or five books. The shipping fee was "**fixed.**" (*It stays the same and doesn't change.*) The cost of one book is also "fixed."

The total amount you would pay would vary according to the number of books you order. Thus the number of books (and the total cost) are the "**variables.**" (*Those numbers can vary or change.*)

Let's create an **equation** that would express the cost of buying books from the internet site. If the flat fee for shipping is \$10, and each book costs \$12, the total cost (C) would be expressed as

$$C = 10 + 12b$$

where \$10 is the fixed cost of shipping, \$12 is the fixed cost of one book, and b is the (variable) number of books bought.

alert As you solve the following questions adapted from previous tests, it would be helpful to determine which values are fixed and which values are variable. As you will see, the test doesn't always simply ask you to solve a problem; it makes you think about how to solve the problem.

Practice Questions

- The total cost for printing a classified advertisement in a local newspaper is made up of a \$30 fee, plus \$0.10 per word.

Which equation below models the relationship where C is the total cost to place the advertisement and w is the number of words? [Circle the letter beside the correct answer.]

a $C = 10 + 0.30w$

c $C = 30 + 10w$

b $C = 10 + 30w$

d $C = 30 + 0.10w$

What are the fixed values in that question? _____

What are the variable values in that question? _____

Answers on page 98.

GRAPHS, EQUATIONS, TABLES

D. Equations

2. Pablo has a cellphone. The relationship between his total monthly cost, C , in dollars, and the number of minutes he uses the phone, t , is represented by the equation $C = 20 + 0.25t$.

Which of the following is not true about this relationship? [Circle the letter beside the correct answer.]

- | | |
|--|--|
| a The cost per minute is \$0.25. | c The total monthly cost for one minute is \$20. |
| b The value of the rate of change is 0.25. | d The graph of the relationship has a C-intercept of 20. |

What are the fixed values in that question? _____

What are the variable values in that question? _____

3. The equation $C = 15n + 100$ represents the relationship between the total cost of a gym membership, C , in dollars, and the number of months of membership, n .

Which statement about the gym membership is true? [Circle the letter beside the correct answer.]

- | | |
|-----------------------------------|--|
| a It has an initial cost of \$15. | c It has an initial cost of \$15 and a fee of \$100 per month. |
| b It costs \$115 per month. | d It has an initial cost of \$100 and a fee of \$15 per month. |

What are the fixed values in that question? _____

What are the variable values in that question? _____

Answers and explanations for page 97-98

- Choose answer d). Cost is $\$30 + 10¢ (0.10) \times$ the number of words.
Fixed: the fee (\$30) and the cost per word (0.10). **Variable:** number of words, total cost.
- Answer c) is not true -- so that's the correct answer. (The question asks "which answer is NOT true.")
Cost for one minute would be \$20.25 (flat fee plus 25¢ for one minute).
All the other answers are true (even if you don't know what "C-intercept" means!).
Fixed: the fee (\$20) and the cost per minute (25¢). **Variable:** number of minutes, total cost.
- Choose answer d). Initial cost is \$100, monthly fee is \$15.
Fixed: the initial cost (\$100) and the fee per month (\$15). **Variable:** number of months, total cost.

GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

Guided Review

Do you remember Jamal and his lawn-cutting job?
He was paid \$15 for every lawn cut.

The equation to show the relation of his Pay (P) to the number of lawns cut was $P = nI$ (n = number of lawns, I = fee per lawn). We can also show this relationship via a table.

Lawns Cut	Pay
1	15
2	30
3	45
4	60

rate of change
= \$15

Lawns Cut	Pay
5	75
6	
7	
8	

You fill in the missing values in this table. Answer on page 104.

Examining the table, you can see that the rate of change (\$15) is the same, no matter how many lawns Jamal cuts.

In particular, the pay for the first lawn cut equals the rate of change, and no more.

alert This is a direct linear variation. (A graph would show a straight line, starting at 0/0.)

Now, let's create a table for Horace. He receives a \$20 flat fee just for coming to work, plus \$12 for every lawn he cuts. The equation would be $P = 20 + 12n$

Horace's pay

Lawns Cut	Pay
1	32
2	44
3	56
4	68

What is different from Jamal's pay is that for doing the first lawn, Horace is paid \$32 (\$20 fee plus \$12 for one lawn), and then he is paid \$12 for every lawn cut after that.

rate of change
= \$12

alert

This is an example of a partial linear variation. (A graph would start at 32 on the y axis, and then slope upward in a straight line, because the rate of change is constant.)



GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

On a separate piece of paper, use tables or a graph to show how many lawns Jamal must cut before he starts to make more money in a day than Horace.

Answer and explanation on page 103.

Here is another question in our Guided Review about Understanding Tables:

Look at the table below, and calculate the fixed shipping cost:

Cost of buying books

Books bought	Cost (\$)
1	25
2	35
3	45
4	55

First, calculate the rate of change. Did you get \$10? (cost of 2 books minus 1 book, or $3 - 2$)

Next, subtract the rate of change from the Cost: First book purchased: $25 - 10 = 15$

The difference of \$15 is the fixed fee, in this case, the cost of shipping.

GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

Guided Review

Here are a couple of wrinkles to show what kind of questions may be asked on your test:

So far, our examples about tables have shown the rate of change -- such as lawns cut -- all in increments of 1 (1, 2, 3, 4 etc.).

alert Tables can also be shown with much greater multiples, such as in the following practice questions.

Practice Questions

Answers on page 104.

1.

A
Cost of
Event

Number of people	Total Cost (\$)
0	0
200	1000
400	2000
600	3000

(Direct Variation)

B
Cost of
Event

Number of people	Total Cost (\$)
0	500
200	1500
400	2500
600	3500

(Partial variation)

A

B

What is the cost per person? _____

What is the rental fee? _____



GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

2. Here is a question that requires you to apply your math logic:

A banquet hall charges a \$1500 rental fee, plus \$25 for each person attending. Which table below shows this relationship?

a Banquet Charges

Number of people	Total Cost (\$)
0	\$1500
5	\$1525

b Banquet Charges

Number of people	Total Cost (\$)
0	\$1500
5	\$3000

c Banquet Charges

Number of people	Total Cost (\$)
5	\$125
250	\$6250

d Banquet Charges

Number of people	Total Cost (\$)
5	\$1625
250	\$7750

3. Solve, using a table. Develop your answer on a rough piece of paper.

Twin Peaks school paid \$2600 for 400 yearbooks. The cost of printing each book was \$4.00. What was the cost of the graphic design that was part of the total price paid?

alert Also, the rate of change can be negative!

4. You are selling chocolate bars for your school band. You start with 80 bars and sell 7 bars per day.

What is the rate of change? _____

Show this relationship on a table.

**Chocolate Bar
Sales Schedule**

Days	Bars remaining
0	80
1	73
2	66
3	59
4	
5	
10	

Fill in the missing values in the table.

alert If this relationship was shown on a graph, the slope line would point downwards, to show a negative change.

5. Ivana is running a marathon of 40 km. She has a pace of 10 km/h.
Create a table showing the distance left to be run.

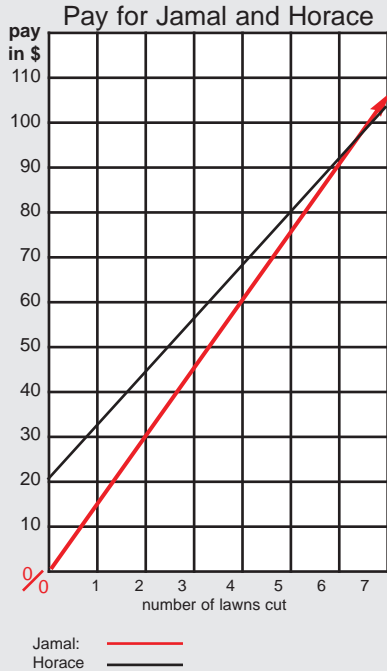
GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

Answers and explanations for pages 99-103

From table at top of page 99: 6 lawns: \$90; 7 lawns: \$105; 8 lawns: \$ 120

page 100:



The red line on the graph for Jamal starts at 0/0, and increases by \$15 for each lawn cut.

The black line for Horace starts at \$20, and increases by \$12 for each lawn cut.

For 7 lawns, Jamal earns \$105, and Horace earns \$104. Jamal must cut 7 lawns before he earns more in a day than Horace.

For each lawn more than seven, Jamal will earn an additional \$3 more than Horace.

You could show the same information by making a table showing side by side the earnings of each worker per lawn.

When you are asked to show either a graph or a table, the table is usually easier to make.

Pay for Jamal and Horace

lawns cut	J \$	H \$
1	15	32
2	30	44
3	45	56
4	60	68
5	75	80
6	90	92
7	105	104
8	120	116
9	135	128

Pages 101-102:

- Table A: cost per person: \$5 Rental fee: \$0
Table B: cost per person: \$5 Rental fee: \$500
- Select answer d). For five people, the cost is \$1500 fee + \$125 (5 x \$25); for 250 people, cost is \$1500 fee + \$6250 (250 x \$25).

- Table at far right. To create table, work upwards. 400 books cost \$2600. Since each book costs \$4, 300 books cost \$400 less. Work your way back to 0 books = cost of graphic design = \$1000.

Page 103:

- Rate of change is -7 (minus 7). Rest of table: 4 days: 52 remain; 5 days: 45 remain. 10 days: 10 remain [80 - (10 x 7)]
[Using integers, it's really 80 + (10 x -7)]

- Table at near right.

hrs	km left
0	40
1	30
2	20
3	10
4	0

# of books	\$
0	1000
100	1400
200	1800
300	2200
400	2600

Guided Review

Let's do a question together, combining graphs and tables.

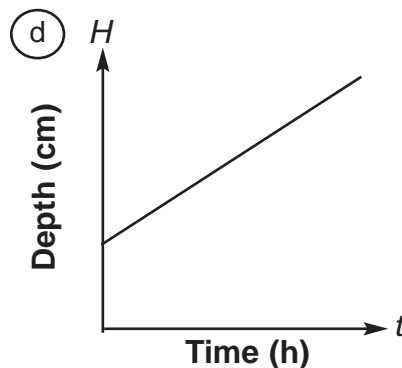
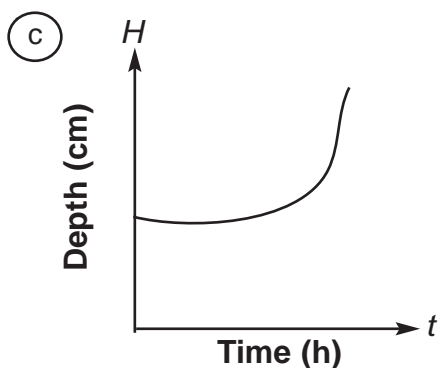
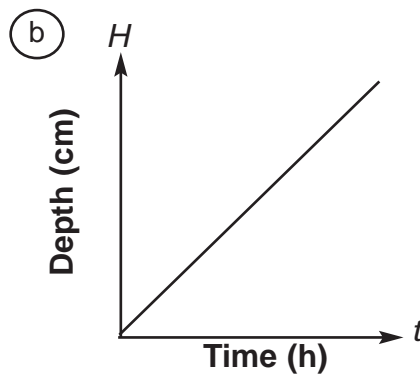
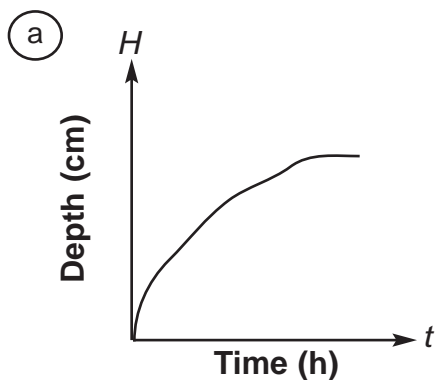
A snowstorm lasts for seven hours. Data is recorded for the depth of snow for the first five hours.

Snowstorm accumulation

Time (h)	Depth of snow (cm)
0	5
1	8
2	11
3	14
4	17
5	20

Which graph below best models the depth of snow during the five hours?

analysis
on next
page



GRAPHS, EQUATIONS, TABLES

E. Understanding Tables

ANALYSIS

What is the main question?

To choose the graph which models the information given in the table on page 105.

Method:

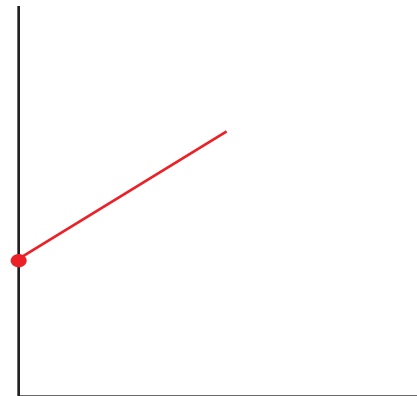
Try making your own rough graph to show the information given in the table. Just start with drawing the two axes, as shown below right.

You need to understand two points presented in the table:

- Is the relationship between time and depth of snow a **direct or partial variation**?*
[Remember that “direct variation” starts at 0/0; partial variation starts at some higher number.]
- Is the relationship between the time and the depth of snow **linear**?*
[Remember, that means the relationship is portrayed by a straight line.]

Therefore:

- Looking at the table, you see that at Time 0 (zero) there was already 5 cm of snow.
So the table presents a **partial variation**.
On your rough graph, place a dot part-way up the y axis.
- The next task is to determine if the relationship between time and depth of snow is linear.
To do that, we must observe the rate of the depth of snow between each one-hour interval listed in the table.



We can see that from 0 to 1 hour, the depth of snow grows from 5 cm to 8 cm. (+3)
From hour 1 to hour 2, the depth increases from 8 cm to 11 cm. (+3)
From hour 2 to hour 3, the depth increases from 11 cm to 14 cm. (+3)
and so on...

Since the rate is always +3, the relationship between time and depth will be shown by a straight line. It is a **linear relationship**. (The relationship is therefore “partial linear.”)
The line will go upwards. Draw a line, starting at the red dot, on the rough graph.

Then look at the graphs you have to choose from in the question on page 105.
Does one of them look like the graph you drew?

Choose graph d.



To make sure that you understand what you have reviewed in this chapter, here is a quick quiz. If you have difficulty with any of these questions, review the “key info” sections in the chapter. If you still don’t understand, it’s time to ask a teacher for help.

1. An online music store provides music that members can download. The store charges a membership fee and a cost per song.

The chart below represents the relationship between the total cost and the number of songs downloaded.

Cost of downloads

Number of songs	Total cost (\$)
10	13
20	16
30	19

Which of the following is **not** true about this relationship?
[Circle the letter beside the correct answer.]

- a It is non-linear.
- b It has an initial cost.
- c It has a constant rate of change.
- d It can be represented by a straight line.

alert

Hints:

Don’t be tricked by the wording of the question: it asks you to find the one sentence that is NOT true.

The best way to get the correct answer is to find the three answers that are correct, and check them off.

The one answer that remains is not true.

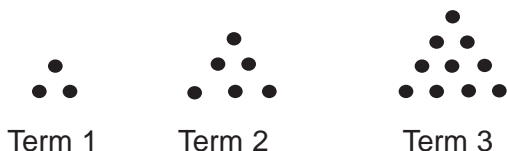
Answer on page 109.

GRAPHS, EQUATIONS, TABLES

F. Checkpoint

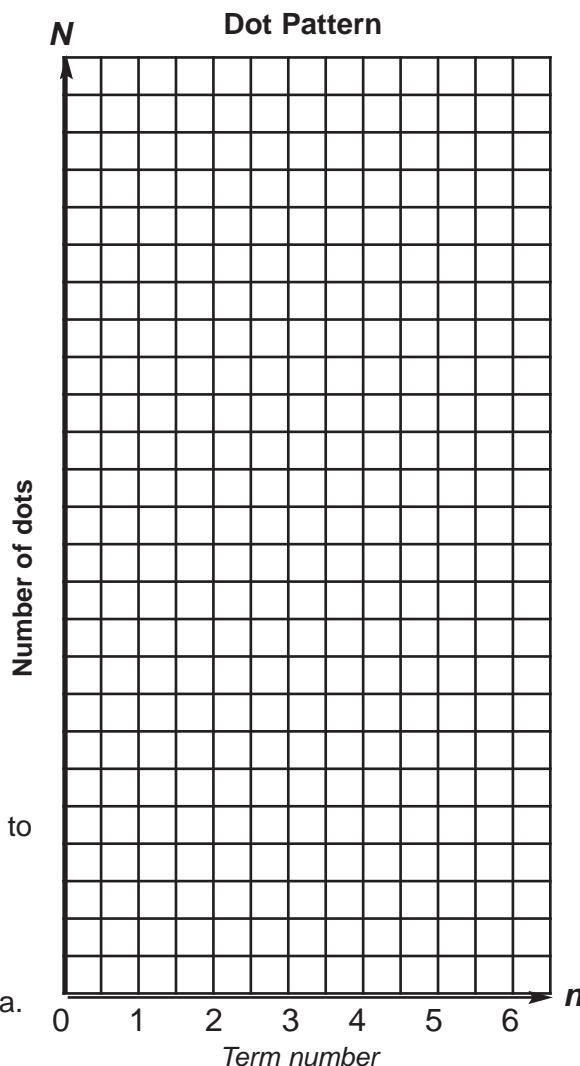
2. Stack It

Juan draws the first three terms of a pattern as shown below.



The pattern continues to grow in the same way. Complete the following table according to the pattern.

Term number, n	Number of dots, N
1	3
2	6
3	
4	
5	
6	



Graph the data from the table on the grid to the right.

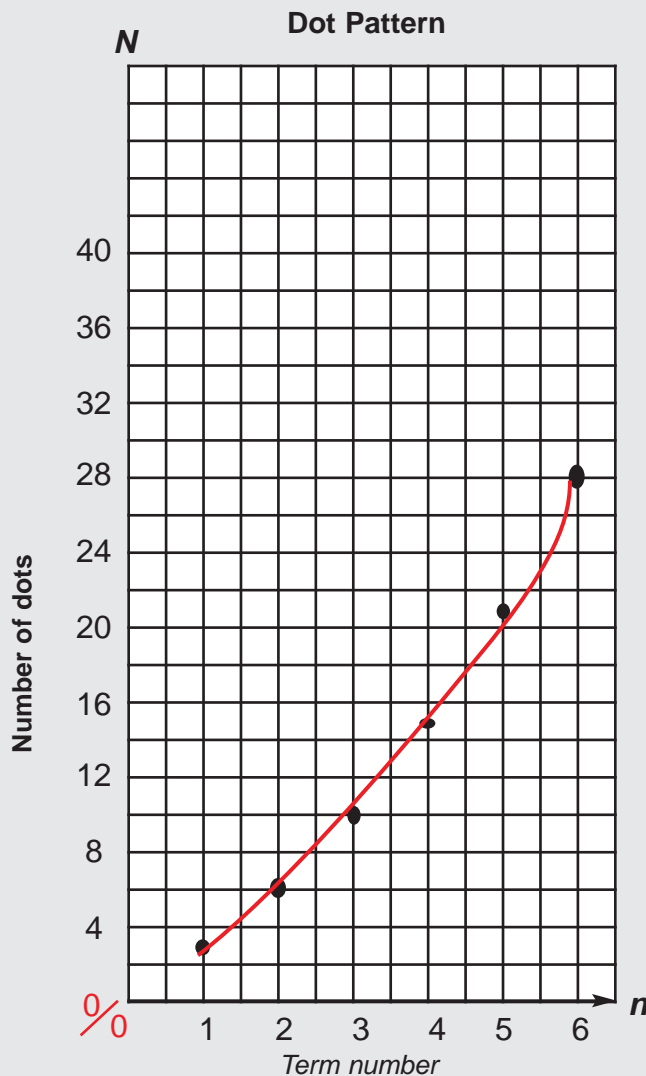
Add a scale for the **N axis**.

Draw a line or curve of best fit for the data.

Answer on page 109.

Answers and explanations for page 107-108

- Choose answer a. The table shows a constant rate of change (\$3 for 10 more songs) -- so answer c is true. Because the rate of change is constant, it can be represented by a straight line on a graph, so answer d is true. Since you pay \$3 for each ten songs, the initial cost would be \$10, so answer b is true. Therefore answer a is the only one that is not true.
- To complete the table, each new row of dots has one more dot than the previous row.
Term number 3: $10(6 + 4)$; term 4: $15(10 + 5)$; term 5: $21(15 + 6)$; term 6: $28(21 + 7)$.



The grid in the question has only 24 rows. The data in the question needs 28 rows in the N-axis. Therefore, Dr. Don is counting each row as showing two dots; then he wrote the numbers for every second row, so the N-axis is shown as 4-8-12... You could show it in other ways.

To plot the number of dots in each term on the graph, make marks on the graph for all six terms, and then join your marks with a line. The line is not straight because the rate of change is not constant. The red line is a curve. The graph shows a non-linear variation.

*This is a difficult question... but it was found on a previous test.
You can solve it by taking it step by step.*

Chapter 5 Ratio, Proportion & Percent



Key Info

Definitions

A. RATIO

alert A **ratio** is a comparison of two quantities (for example, the comparison between 2 and 5).

A ratio can be written in three different ways:

- as a fraction: $\frac{2}{5}$ or $2/5$ *It's probably easiest to think of a ratio as a fraction.*
- as two quantities separated by a colon: $2:5$
- as two quantities separated by the word "to": 2 to 5
(“The ratio of sugar to flour in that recipe is 2 to 5.”)

Ratios should always be reduced to lowest terms. Don't show it as $4/10$ or $16/40$; write $2/5$!

Some books define ratio as “the quotient of two quantities, arrived at by dividing one by the other.”

That complicated definition might help you realize that *every* fraction is a division question!
The fraction $2/5$ means “2 divided by 5.”
The answer in decimals is .4.

Guided Review

1. Your school's entire Grade Nine class of 150 students has 70 boys.
What is the ratio of girls to the whole class?

Dr. Don has underlined the key words.

alert You need to create a ratio of “girls to whole class” (not boys to girls, or boys to class).

You are given the number of boys and the total number in the class, but you need the number of girls before you can build the ratio.

That's easy, right?

You simply subtract the number of boys from the total class to find the number of girls.

$150 - 70 = 80$. There are 80 girls in the class.



RATIO, PROPORTION, PERCENT

A. Ratio and Proportion

The ratio of girls to the whole class is

$$\frac{\text{girls}}{\text{class}} = \frac{80}{150}$$

When it's showed as a fraction like this, you should **always reduce the fraction to lowest terms** by dividing top and bottom by the same number until you can't do it any more.

$$\frac{80}{150} = \frac{8}{15}$$

The ratio of girls to the class can also be expressed as 8:15 or 8 to 15.

Practice Question

Now you try one:

A magician has a collection of red and blue magic balls in a bag. Of the 15 magic balls, 5 are red. What is the ratio of red balls to blue balls? Justify your answer.

alert

Dr. Don's hint

First underline the key words in the question.

Then solve the problem.

Answer is found below.

Answer and explanation

Underline key words: A magician has a collection of red and blue magic balls in a bag. Of the 15 magic balls, 5 are red. What is the ratio of red balls to blue balls?

Step one: Find how many balls are blue: $15 - 5 = 10$.

Answer: ratio of red:blue = 5:10, or in lowest terms 1:2 or 1/2 or 1 to 2.

RATIO, PROPORTION, PERCENT

A. Ratio and Proportion



Key Info

B. PROPORTION

alert

A **proportion** is a statement of equality between two ratios. In other words, it's an equation showing that two fractions are equal.

It might be obvious, like $\frac{5}{6} = \frac{10}{12}$,

or it might involve an unknown number that you have to find, like $\frac{5}{6} = \frac{a}{36}$

Guided Review

Definitions help some students to understand mathematical concepts, but other students may find that a real-life example will work best. How about buying pizza?

1. Pizza in your school cafeteria costs \$5 for two slices.

alert

The relationship between the number of slices (2) and the cost (\$5) is a **ratio**. It is a comparison of two quantities, as the definition said. The ratio of slices to cost can be written as $2/5$, $2:5$ or 2 to 5.

If you're with a hungry friend, you might want to buy six slices of pizza.

alert

You can probably figure out quickly how much six slices would cost, but we want to show you how to figure it out mathematically as a **proportion** (an equation showing that two fractions are equal).

Dr. Don thinks you should write the values that you are comparing in words before you write the numbers in the proportion, like this:

$$\frac{\text{slices}}{\text{cost}} \quad \frac{2}{5} = \frac{6}{x}$$

You can easily solve such an equation by realizing that the two quantities (slices and cost) must change at the same rate.

alert

If you multiply the top of the ratio by a certain number, you must multiply the bottom of the ratio by the same number.

On the top of the ratios (referring to the number of slices of pizza), what do you do to the 2 to make it become 6? You multiply by 3.

Therefore, you must multiply the number on the bottom of the ratio (\$5) by the same number, to find x , the cost of six slices: \$15 is the answer, of course.



In this kind of problem you are given three values and asked to find the fourth.

RATIO, PROPORTION, PERCENT

A. Ratio and Proportion

2. Here is another question for us to do together -- three different ways:

If 2 litres of ice cream cost \$5.40, how much would 5 litres cost?

Method #1: Since you know the cost of 2 litres, find the cost of one litre: $\$5.40 \div 2 = \2.70
Then multiply the cost of 1L by 5 to get the cost of 5 litres: $\$2.70 \times 5 = \13.50 .

Method #2: Since 5 litres is 2.5 times as much as 2 litres ($5 \div 2 = 2.5$),
the cost of 5 litres will be 2.5 times the cost of 2 litres. $\$5.40 \times 2.5 = \13.50 .

Method #3: Build a proportion, and solve by cross-multiplying:

$$\begin{array}{l} \frac{\text{litres}}{\text{cost}} \quad \frac{2}{5.40} = \frac{5}{x} \\ \frac{2}{5.40} \quad \frac{5}{x} \\ 2x = 5.40 \times 5 \\ 2x = 27.00 \\ x = 13.50 \end{array}$$

5 litres of ice cream would cost \$13.50.

Why does cross-multiplying work?

In our example, when you multiply $2 \times x$, you're really multiplying both sides of the equation by x .
When you multiply $5/x$ by x , the answer is 5 (the x disappears).

When you multiply 5.40×5 , you're really multiplying both sides of the equation by 5.40.
When you multiply $2/5.40 \times 5.40$, the answer is 2.

The result: $2x = 5.40 \times 5$

3. Let's try another proportion question together, solving by cross-multiplying.

If a car runs 100 km on 9 litres of fuel, how far will it travel on 27 litres?

What you need to find is how many km can it travel on 27 litres. *Underline it!*

Use x as the unknown number of kilometres.

Then, build your proportion: $\frac{\text{litres}}{\text{km}} \quad \frac{9}{100} = \frac{27}{x}$

Cross-multiply:
$$\begin{array}{l} 9x = 27 \times 100 \\ 9x = 2700 \\ x = 300 \end{array}$$

You can travel 300 km on 27 L of gas.

alert Dr. Don's notes:

Most textbooks recommend that you use the "cross-multiplication" method to solve proportion problems, even for more complex questions.

Keep equal signs under each other when solving equations.

RATIO, PROPORTION, PERCENT

A. Ratio and Proportion

Practice Questions

1. Consider the following proportion:

$$\frac{5}{6} = \frac{a}{36}$$

What is the value of a in the proportion?

- a. 6
- b. 5
- c. 11
- d. 30

2. The dimensions of a field are in a 4:5 ratio. If the width is 32 m, what is the length?

- a. 8 m
- b. 20 m
- c. 32 m
- d. 40 m

The next question is more complex, because it asks you to take one more step after you get the first answer, but it will be easily solved by building a proportion -- and using some math logic.

3. You are mixing some sportsade powder with water for the use of the basketball team.

The directions require 2 scoops of powder for 1200 mL of water.

Your sister has put 8 scoops of powder into 4000 mL of water.

How much more water needs to be added to make the mixture taste right?

Show your work here.



Answers are found on the following page.

RATIO, PROPORTION, PERCENT

A. Ratio and Proportion

Answers and explanations for page 114.

1. Cross-multiply: $5 \times 36 = 6 \times a$

$$180 = 6a$$

$$30 = a \quad \text{Select answer d.}$$

2. Build a proportion

$$\frac{\text{Width}}{\text{Length}} = \frac{4}{5} = \frac{32}{x}$$

Cross-multiply: $4 \times x = 5 \times 32$

$$4x = 160$$

$$x = 40 \quad \text{Select answer d.}$$

3. **Method #1:** Let x be the amount of water needed by the recipe for 8 scoops.

Build a proportion.

$$\frac{\text{scoops}}{\text{mL water}} = \frac{2}{1200} = \frac{8}{x}$$

Cross-multiply: $2x = 9600$

$$x = 4800$$

alert

Here's the logic: the recipe calls for 4800 mL of water. Your sister has put 8 scoops into 4000 mL. How much more water is needed? $4800 - 4000 = 800$ mL

[We're not talking about huge amounts here: 4000 mL is four litres. A measuring cup is 250 mL, so you would add a bit more than 3 cups.]

Method #2 still uses proportion, but now let y be the amount of water that must be added to the 4000 mL that are already there.

$$\frac{\text{scoops}}{\text{mL water}} = \frac{2}{1200} = \frac{8}{(4000 + y)}$$

Cross-multiply: $2(4000 + y) = 8 \times 1200$

$$8000 + 2y = 9600$$

(subtract 8000 from both sides of the equation: $8000 - 8000 + 2y = 9600 - 8000$)

$$2y = 1600$$

$$y = 800$$

So again, 800 mL of water must be added to make 8 scoops of powder taste right.

Be careful! Avoid careless mistakes!

Dr. Don's strategy reminds you:
always underline the key elements
in mathematical problems.

RATIO, PROPORTION, PERCENT

A. Ratio and Proportion



Key Info

B. Percent (Decimal method)

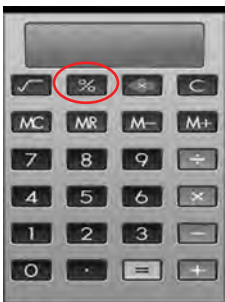
Every percent is a fraction with 100 as its denominator.
45% means $45/100$ (that means “forty-five hundredths”).

Every fraction is a division question. $45/100$ means $45 \div 100$. The answer is .45, or 45%.

alert

When you’re using decimals, you can find “hundredths” by looking at **the first two numbers after the decimal point.**

- .25 means “25 hundredths” or $25/100$ or 25%.
- When there is only one number after the decimal point, add a zero: $.4 = .40 = 40\%$
- To express a percent less than 10%, insert a 0. $.4\% = .04$
- When there are more than two numbers after the decimal point, round it off to two digits
 $> .252 = 25\%$ or 25.2% ; $.257 = 26\%$ or 25.7%
- When there is a number before the decimal point, you can still get the percent by using the number before the decimal and the first two numbers after the decimal point.
 $1.13 = 113\%$ ($100\% + 13\%$ -- that’s how to calculate the total cost of an item with 13% sales tax)



On the official test, you are permitted to use your calculator, and it is especially useful in doing percent questions.

alert

**Dr. Don’s advice is “DON’T use the % key on the calculator!”
Do it all -- on your calculator -- with decimals!**

Different calculator manufacturers set the % key to work in different ways. Unless you study the operator’s manual for your particular calculator, or else do a lot of experiments with it, you can never be sure what the % key will do.

On the internet, if you ask how to use the % key on a calculator, many mathematicians and engineers answer that they never use the % key on their calculator.

The rest of this section on percent will tell you how to answer percent questions with your calculator using decimal points, multiplication and division.

RATIO, PROPORTION, PERCENT

B. Percent

alert When you use the decimal method, *there are only two types of questions* about percent, but various ways of wording questions sometimes lead to confusion.

Type 1. What percent is x of y ? or What percent of y is x ? DIVIDE!

Type 2. What is $x\%$ of y ? MULTIPLY!

Always look for these two basic types of questions, and you will be able to answer any question they ask about percent.

Guided Review

Type 1. What percent is x of y ? or What percent of y is x ? DIVIDE!

alert The two ways of asking Type 1 questions mean the same thing. Though “is” and “of” appear in a different order, both phrases mean exactly the same.

This type of question always has the words “what percent”?

What percent of 40 is 30? or What percent is 30 of 40?

It's the question that you ask when you get 30 out of 40 on a test: What percent did I get?

You can get the answer to this type of question using a calculator by **dividing**.

$\frac{30}{40}$ means $30 \div 40$. The answer is .75.

The first two numbers after the decimal tell you the percent: 30 is 75% of 40.

Every time the question asks “what percent?” think of it as if you got $\frac{30}{40}$ on a test.

Every fraction is a division question.

Enter $30 \div 40$, and the first two numbers after the decimal point tell you the percent.

alert *It's important to enter the numbers into your calculator in the right order.*

When the question says “What percent is 30 of 40?” or “What percent of 40 is 30?”

think “is over of” $\frac{\text{is}}{\text{of}}$. What percent $\frac{\text{is}}{\text{of}}$ $\frac{30}{40} = 30 \div 40 = .75 = 75\%$

RATIO, PROPORTION, PERCENT

B. Percent (decimal method)

Practice Questions

Take your calculator, and try a few. Show your work.

1. What percent is 16 of 64? _____
2. What percent is 40 of 50? _____
3. What percent of 150 is 100? _____
4. What percent of 70 is 35? _____
5. What percent is 9 of 81? _____

alert Hint:

Remember "is over of"

is
of

Guided Review

Type 2. What is $x\%$ of y ? **MULTIPLY!**

alert In Type 2 questions, you are *given* the percent.
[In type 1, you are asked to *find* the percent.]

This type of question will say "What is 40% of 85?" or "What is 4% of \$50?"
In those questions, you are given the percent (40%, 4%) and asked to find that percent of a certain number.

The way to get the answers using a calculator: **multiply!**

> What is 40% of 85?

Multiply .40 (= 40% -- first two numbers after the decimal point) by 85. Answer is 34.
40% of 85 is 34, or 34 is 40% of 85.

> What is 4% of \$50?

Multiply .04 by \$50 (that zero is important: the first two numbers after the decimal point tell the percent. .04 is 4%; .4 is 40%). Answer is 2. 4% of \$50 is \$2.

Answers and explanations for qq 1-5 above

1. $16 \div 64 = 0.25$ **First two numbers after the decimal:** 25%
2. $40 \div 50 = 0.8$ 80%
3. $100 \div 150 = 0.6666$ 67% or 66.67%
Remember is/of when you enter numbers into the calculator!!!
4. $35/70$ or $35 \div 70 = 0.5$ 50% **Be sure: two numbers after the decimal.**
5. $9 \div 81 = 0.11111$ 11% or 11.1%